Finding Subsets of Groups that Hold Measurement Invariance

a simple method and a Shiny app

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ESRA Conference Zagreb
2019

Motivation

Measurement invariance

> *"The general question of invariance of measurement is one of whether or not, under different conditions of observing and studying phenomena, measurement operations yield measures of the same attribute"*

(Horn and McArdle, 1992)

Procedures to assess measurement invariance with confirmatory factor models

Run several nested multiple groups CFA models with a growing set of constraints, usually:

- configural (overall similarity of structures);
- metric (equality of loadings);
- scalar (equality of loadings and intercepts).

and compare their model fit, that should be approximately the same.

When invariance is not supported: 1. find a subset of parameters which are invariant

Given the model is specified correctly, several options:

- partial invariance (relax some constraints, but not less than two per each factor - Byrne, Shavelson, & Muthen, 1989);
- approximate invariance (relax strict equality of parameters, Bayesian zero priors on differences);

When invariance is not supported: 2. find a subset of groups

- repeatedly re-run an MGCFA model with different subsets of groups;
- alignment method (Muthen & Asparouhov, 2013, 2014a, 2014b), minimizing non-invariance by finding convenient factor means and variances (available only in Mplus), however:
  - *"The assumption of the alignment method is that a majority of the parameters are invariant and a minority of the parameters are non-invariant." (Muthen & Asparouhov, 2013)*
  - majority of groups have invariant parameters;
  - there are many groups.

Case 1. Clusters of Groups

True model

20 groups (with 500 observations) with 4 clusters

```
F1 =~ v1  + v2  + v3  + v4;
F2 =~ v11 + v12 + v13 + v14;
```

Parameter	Gr_1to5	Gr_6_10	Gr_11_15	Gr_16_20
F1 by v1	1	1.0	1.0	1.0
F1 by v2	1	0.4	0.4	0.4
F1 by v3	1	0.3	0.7	0.1
F1 by v4	1	0.2	0.7	1.0
F2 by v11	1	1.0	1.0	1.0
F2 by v12	1	0.4	0.4	0.4
F2 by v13	1	0.3	0.7	0.1
F2 by v14	1	0.2	0.7	1.0

Conventional tests of invariance: there is no metric invariance

```
Globam MI testt:

Chi Square Difference Test
                Df Chisq Chisq diff Df diff Pr(>Chisq)
fit.configural 380 379.29
fit.loadings   494 2188.84    1809.55     114     0.000 ***

Fit measures:
                 CFI RMSEA DIFF.CFI   DIFF.RMSEA
fit.configural 1.000 0.000
fit.loadings   0.850 0.083     0.150        0.083
```

Modification indices: >50 large ones, hard to captrue the pattern

	lhs	op	rhs	block	mi	epc
1267	v11	~~	v14	16	101.139	0.833
1411	v11	~~	v14	20	86.267	0.727
1317	v1	~~	v4	18	85.138	0.704
1339	v11	~~	v14	18	83.521	0.779
1375	v11	~~	v14	19	78.240	0.715
1353	v1	~~	v4	19	77.748	0.719
1389	v1	~~	v4	20	73.318	0.715
1281	v1	~~	v4	17	70.707	0.688
1303	v11	~~	v14	17	50.789	0.581
1245	v1	~~	v4	16	37.319	0.491
498	f2	=~	v11	18	37.038	1.018

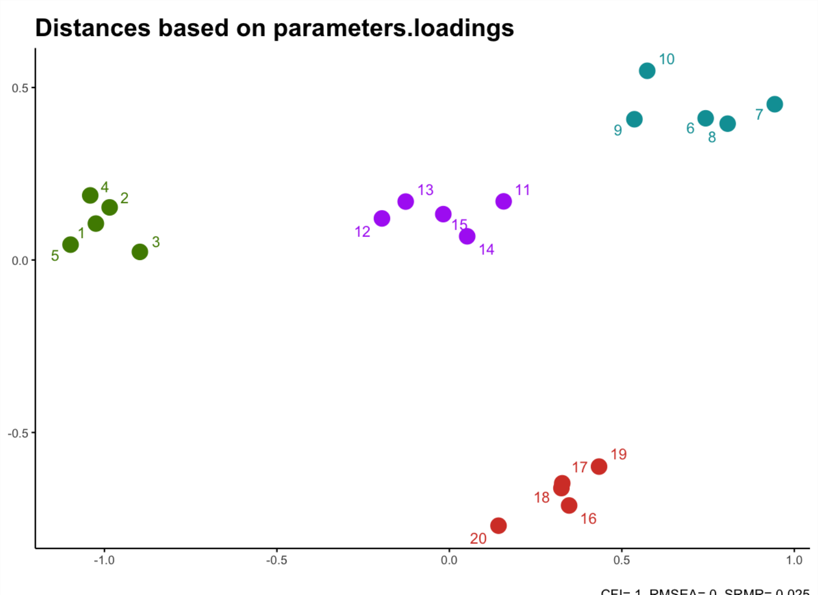
Alignment in Mplus (fixed mode): right direction, but

Loadings for F1 V1 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
V2 **(1) (2) (3) (4) (5)** 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
V3 **(1) (2) (3) (4) (5)** 6 7 8 9 10 **(11) (12) (13) (14) (15)** 16 17 18 19 20
V4 1 2 3 4 5 **(6) (7) (8) (9) (10)** 11 12 13 14 15 16 17 18 19 20

Loadings for F2
V11 1 2 **(3) (4) (5)** 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
V12 **(1) (2) (3) (4) (5)** 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
V13 **(1) (2) (3) (4) (5)** 6 7 8 9 10 **(11) (12) (13) (14) (15)** 16 **(17)** 18 19 20
V14 1 2 3 4 5 **(6) (7) (8) (9) (10)** 11 12 13 14 15 16 17 18 19 20

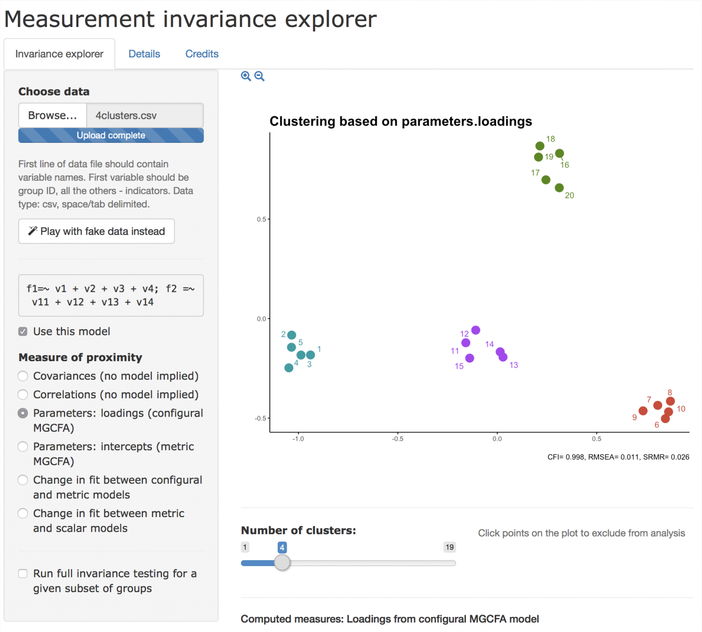
Average Invariance index: 0.383

It would be nice to represent differences in measurement models in terms of distances on a plot (Ignacz, 2017)



Introducing Measurement Invariance Explorer (MIE)

MIE interface



MIE logic

For user: User uploads data, specifies a model, chooses a measure, excludes/includes groups, looks for possible clusters and/or outlier groups.

Internally:

- Reads data ->
- (Fits models in `lavaan`) ->
- Extracts measures ->
- Subsets measures ->
- Computes distance matrix (`dist`) ->
- Finds two-dimensional projection (`cmdscale`) ->
- Computes kmeans clusters based on measures ->
- Plots using clusters for coloring points.

Measures of "invariance distance": no model implied

- **Covariances (no model implied)** Commonly used multidimensional scaling of all available indicators. Two dimensions are extracted.
- **Correlations (no model implied)** After applying Fisher's z transformation, the distances are computed, sent to MDS and plotted.
  - If the model fits data well, corrleations/covariances and model parameters should differ across groups in a similar way.

Measures of “invariance distance”: parameters in a global model

- **Parameters: loadings (configural MGCFA)** A single multiple group confirmatory factor analysis with non-constrained factor loadings and intercepts. It extracts loadings, and uses them to compute distance matrix, which is then scaled and plotted.
- **Parameters: intercepts (metric MGCFA)** Analogous to previous one, but loadings are constrained and free intercepts are used as a measure of distance between groups.

Measures of “invariance distance”: change in fit indices of pairwise models

- **Change in fit indices from configural to metric model**
    1. Configural and metric MGCFA models are fitted to subsamples of every possible pair of groups.
    2. Global fit indices are extracted and their change between the two models is computed, they reflect “invariance distances” between each pair of groups.
    3. Without further transformations, CFI, RMSEA, or SRMR for each pair of groups are used to compute two-dimensional scaling and plot the group points.
- **Change in fit indices from metric to scalar model**
  Analogous to previous one, only metric and scalar models are fitted to the pairs of groups.

Problematic point: CFA, RMSEA, and SRMR, as well as their decreases do not have known distributions.

How to speed up?

Global MGCFA is fitted only once, and while a user tries other options, the extracted measures are stored locally.

When pairwise fit index decrease is used, the corresponding models are computed for each pair of groups only once. They are stored locally during the whole session.

Case 2. Outlier Groups

True model

6 groups (with 500 observations)

```
F1 =~ v1  + v2  + v3  + v4;
F2 =~ v11 + v12 + v13 + v14;
```

Parameter	Gr.1	Gr.2	Gr.3	Gr.4	Outlier.Gr.5	Outlier.Gr.6
F1 by v1	1	1	1	1	1.0	1.0
F1 by v2	1	1	1	1	0.6	0.7
F1 by v3	1	1	1	1	0.5	0.4
F1 by v4	1	1	1	1	0.4	0.6
F2 by v11	1	1	1	1	1.0	1.0
F2 by v12	1	1	1	1	0.6	0.7
F2 by v13	1	1	1	1	0.5	0.4
F2 by v14	1	1	1	1	0.4	0.6

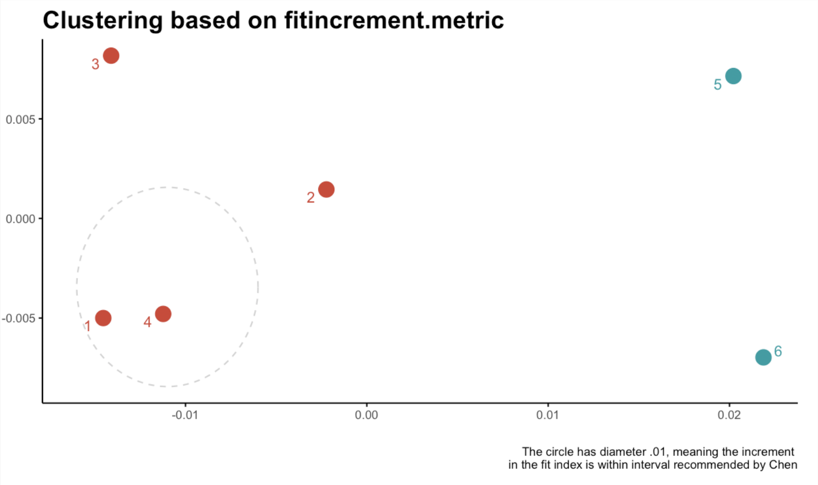
Conventional tests: no metric invariance

```
Global MI output:
 Chi Square Difference Test

           Df Chisq Chisq diff Df diff Pr(>Chisq)
Configural 114 107.36
Metric     144 297.81    190.447      30    < 2e-16 ***

             CFI  DIFF.CFI  RMSEA  DIFF.RMSEA
Configural 1.000            0.000
Metric     0.989    -0.011  0.031       0.031
```

Alignment (fixed mode): something is not right

```
Loadings for F1
   V1          1 2 3 4 5 6
   V2          1 2 3 4 5 6
   V3          1 2 3 4 5 6
   V4          1 (2) (3) (4) 5 6
 Loadings for F2
   V11         1 2 3 4 5 6
   V12         1 2 3 4 5 6
   V13         1 2 3 4 5 (6)
   V14         1 2 3 4 5 (6)
 Groups With Significantly Smaller Factor F1 Mean:
    4 3 2
 Average Invariance index:       0.489
```

MI explorer (plot based on RMSEA increments): clear detection of outliers



Case 3. Real-data example

Mental health scale from European Social Survey, round 7

Model:

```
mhealth =~  fltdpr + flteeff + slprl  + fltlnl+
              fltsd + cldgng +
              wrhpp + enjlf;

wrhpp ~~   enjlf; # reverse coded
```

1 factor, 21 countries, ~2000 observation in each.

Conventional tests

```
Chi Square Difference Test
                Df   Chisq Chisq diff Df diff Pr(>Chisq)
fit.configural 399  4292.8
fit.loadings   539  5796.2     1503.5     140  0.000 ***

Fit measures:
                 cfi rmsea cfi.delta rmsea.delta
fit.configural 0.960 0.072
fit.loadings   0.946 0.072     0.014       0.000
```

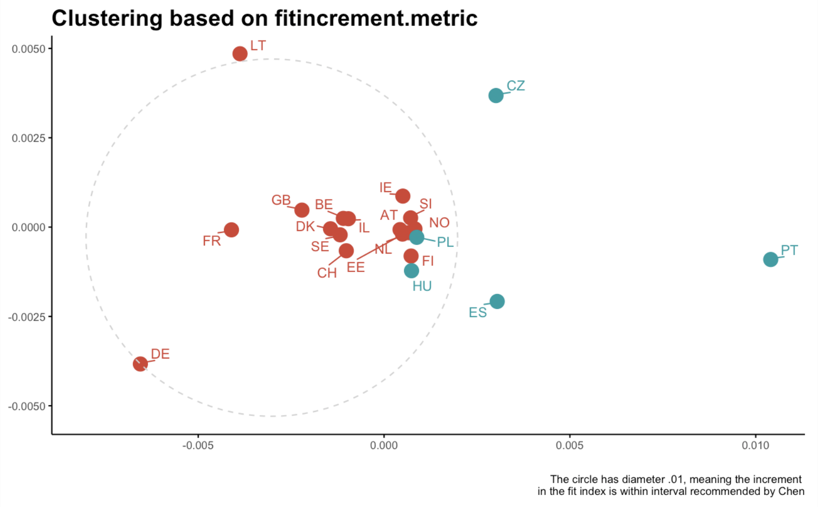
Alignment (fixed mode): few suggestions

5 non-invariant loadings for groups: “CZ” “LT” “PT”

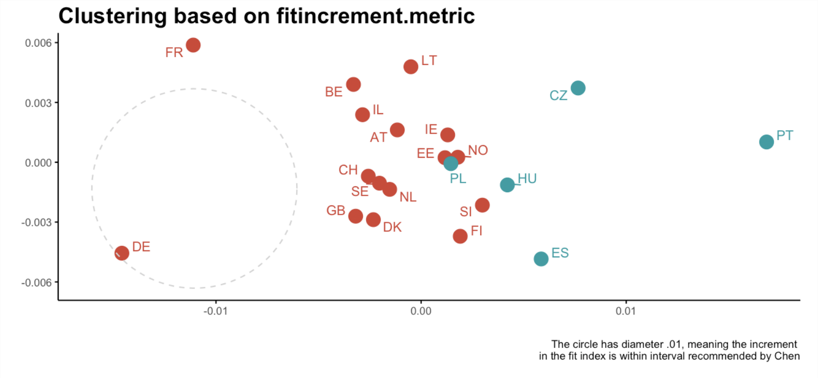
4 non-invariant loadings for groups: “DE”

3 non-invarinat loadings for groups: “BE” “ES” “HU”

MI Explorer - RMSEA increments from configural to metric model



MI Explorer - CFI decreases from configural to metric model

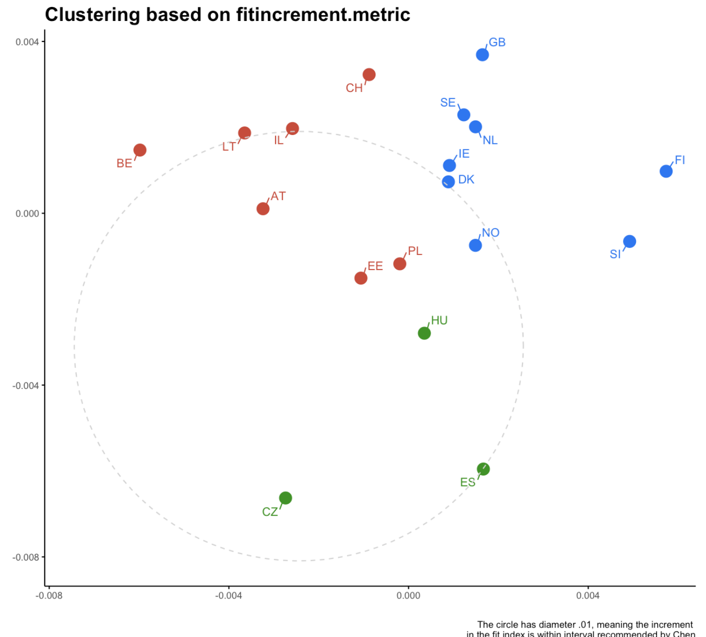


MI Explorer - check the CFI decreases

Computed measures: CFI difference between configural and metric models

Group.1	Group.2	configural	metric	difference
DE	PT	0.964	0.933	0.032
FR	PT	0.957	0.93	0.027
CZ	DE	0.961	0.938	0.023
GB	PT	0.965	0.944	0.021
ES	FR	0.957	0.936	0.02
CZ	FR	0.954	0.935	0.019
BE	PT	0.969	0.952	0.017
DE	SI	0.956	0.939	0.017

MI Explorer - CFI decreases after dropping DE, FR, and PT



MI Explorer - omnibus tests after exclusion of three groups

```
Chi Square Difference Test

                 Df Chisq Chisq diff Df diff Pr(>Chisq)
fit.configural 342 3643.2
fit.loadings   461 4607.6      964.4     119   0.000 ***

Fit measures:
                  cfi rmsea cfi.delta rmsea.delta
fit.configural 0.961 0.073
fit.loadings   0.951 0.070     0.010       0.003
```

Concluding remarks

Use of MI explorer

- effectively suggests clusters of measurement model groups;
- identifies outlier groups (in terms of measurement model);
- integrates different (soft and strict) crietria of measurement invariance on one page;
- interactive: updates models with a single click;
- fast: avoids excessive computations (+ better run locally);
- has a potential to integrate more different approaches in one (graphical) framework.

Misuse of MI explorer

- testing hypotheses (refer to “explorer”);
- looking for model misspecifications.

Compared to finite mixture models

Models that look for latent classes based on factor structures:

De Roover et al. (2017; 2019.

They do better job identifying subsets of groups.

But:

- can be extremely complex, hard to specify, and slow;
- mostly available in propriatery software;
- provide unnecessary precision.

Access

Standalone Shiny app

Demo online:

http://apps.maksimrudnev.com:3838/MIE/

or you run it locally from gitHub:

```
shiny::runGitHub("maksimrudnev/MIE")
```

MIE package

```
devtools::install_github("maksimrudnev/MIE.package",
dependencies = TRUE)
```

For example:

```
runMIE(model = "F =~ impfree + iphlppl + ipsuces + ipstrgv"
       data = ess8,
       group = "country"
       )
```

Other functions in MIE package

- `globalMI` Test for measureent invariance in three steps;
- `computeCorrelation` Computes and formats Fisher's transformed correlation matrices;
- `computeCovariance` Computes and formats covariance matrices;
- `MGCFAparameters` Computes MGCFA models for all available groups in the data;
- `pairwiseFit` Compute the MGCFA model for a given pair of groups;
- `incrementalFit` Run pairwise models and compute decrease in fit;
- `plotDistances` Plot distances using computed measures of distance.

Thank you!

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http://github.com/maksimrudnev/MIE.package